Use ORIGAMI to introduce geometry and algebra ideas.

Origami
(from ori meaning folding, and kami meaning paper)

Origami is the Japanese art of paper folding.

It started in the 17th century AD and was popularized in the mid-1900s.

In 1930 Akira Yoshizawa, a Japanese origami artist/writer, comes up with a way of illustrating the steps. This revitalized origami throughout the world.

Once it has been demonstrated that algebra can be taught three or even thirty ways, it will be malpractice to declare “Johnny could not learn algebra my way—bring me another child.”

Howard Gardner ([1], p. 57)
In origami the goal is to turn a flat piece of paper into a three dimensional sculpture.

Cutting and gluing are not acceptable.

Traditionally a square sheet of paper is used (but it is okay to break the rule!)

Famous Names in Origami

Akira Yoshizawa
Japanese Origami Artist/Writer
(1911-2005)

Tomoko Fuse
Japanese Origami Artist/Writer
(1951-)

Robert J. Lang
American Physicist/Mathematician/Origami Artist
(1950-)

Erik Demaine
Canadian-American Computer Scientist/ Mathematician/Origami Artist
(1981-)
Can origami save life?

Figure 1. Buckling pattern of a thin-walled tube under torsion.

Figure 2. One of the folding patterns with helical folds for tubular stents. (a) It is fully expanded and (b) completely folded.

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One of the goals of the contemporary reform movement is to make many of the abstract ideas of mathematics concrete (whenever possible).

Origami helps students engage in spatial visualization, and communicate better.

**Origami star.**

PART I:

**Making a STAR**

Directions:

It takes 8 pieces of paper to complete the STAR.

Stage 1

Fold along dotted lines.
There are three folds shown here.
Stage 2

Fold two corners down. Use the midpoints of the sides as a guide.

Stage 3

Turn the paper over so that it looks like this. Then press down on point A as you fold segments BC and DE together. The result is a parallelogram.
Stage 4

Once you have folded 8 parallelograms, connect them by placing one inside the fold of another. To make the connection, fold the points of one parallelogram.

Finally, slide the opposite sides to form the star.

**PART II:**

**VOCABULARY:**

- DISTANCE
- MIDPOINT
- DIAGONAL
- INTERSECT
- ALTITUDE
- CONGRUENT
- TRIANGLES
- LINE OF SYMMETRY
- SLOPE
- PERPENDICULAR
- PARALLELOGRAM
- OPPOSITE SIDES
- OPPOSITE ANGLES
- OCTAGON
- POLYGON
Discussion questions:

Stage 1

Describe symmetry with stage 1? Think about symmetry with respect to a point or a side. Use rotation, reflection, or translation to describe a transformation that carries part of the figure onto another.

(G-CO-5)

Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Stage 2

How do we know that the dotted lines connect midpoints in the figure?

Stage 3

How can the following formulas help to show that the figure is a parallelogram?

1) Distance 2) Midpoint 3) Slope

Use graph paper and the formulas above to show that the quadrilateral is a parallelogram
Part III:

Area, Pythagorean Theorem, and Special Right Triangles (a little algebra)

Look at stage 1.

Find the area of the square in as many ways as possible.

A. length x width
B. half x diagonal₁ x diagonal₂
C. 4 triangles
D. Half x apothem x perimeter
E. 8 triangles
F. 2 triangles

Give a general formula for the area of a square that uses the following information only:

1. only the lengths of the sides
2. only the lengths of the diagonals
3. only the apothem
4. only the radius

Explain how 1 and 4 are related.

Critical questions:

What algebra skills are used?
What vocabulary words are used?
What special triangles are used?

For each part of the exercise above, make a list of CCSS that apply.
Mathematics and Origami
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The Optimal Origami Box
Math in Action
Feb. 22, 2007

Shelly Smith
Grand Valley State University
http://faculty.gvsu.edu/smithshe

A hands-on activity where students fold origami boxes with varying heights to explore modeling and optimization. You can download this activity from my website and modify it for use in your class.

Adapted from Unfolding Mathematics with Origami Boxes, by Arnold Tubis and Crystal Mills.

Materials:
• Folding instructions: Wikipedia
• Square origami (scrapbooking) paper
• Rulers
• graphing calculators
Magazine Box

1.

2.

3.

4.

5. Fold hems and open up.

Magazine Box from folding procedure based on easy crease “landmarks”

6. The Magazine Box.
Magazine box crease pattern.

\[
L = l + 2h + 2w_{\text{hem}}
\]

\[
W = w + 2h
\]
Question 1:

What size paper is required for folding a Magazine Box of length 4", width 3", height 2", and a hem width of 1"?

Question 2:

A Magazine Box with:

Length = width = twice the height is folded from an 8 ½” x 11" sheet of paper.

Determine the hem width.
Some Mathematical Concepts and Techniques Involved in Studies of the Generalized Masu Designs

- Algebraic Equations
- Angles
- Area and Volume
- Arithmetic
- Bisection (line, angle)
- Calculator Math
- Comparison of theoretical and actual measure or box parameters
- Congruence (verified by folding)
- Fractions and ratios
- Graphical analysis
- Maxima/minima of box parameters
- Percent error
- Polygons (triangles, rectangles, . . .)
- Pythagorean theorem
- Rectangular solid
- Spatial visualization
- Symmetry

No longer is the purpose of education is simply to pick out those students who are intelligent, on one or another definition, and give them special access to higher education. Rather, the purpose of education now is to educate an entire population, for we cannot afford to waste any minds.

Howard Gardner ([1], p. 238)
The Optimal Origami Box

In this activity, we will fold origami boxes with varying height, and determine which height will give us a box with the largest possible volume.

Folding diagram from Wikipedia entry for Japanese masu


<table>
<thead>
<tr>
<th>Step 01</th>
<th>Step 02</th>
<th>Step 03</th>
<th>Step 04</th>
<th>Step 05</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Step 01" /></td>
<td><img src="image2.png" alt="Step 02" /></td>
<td><img src="image3.png" alt="Step 03" /></td>
<td><img src="image4.png" alt="Step 04" /></td>
<td><img src="image5.png" alt="Step 05" /></td>
</tr>
</tbody>
</table>

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>Crease and Return</td>
<td>Fold tops to centre, this is called a blintz fold after a Jewish pastry</td>
<td>Fold sides to centre and return</td>
<td>Open two corners</td>
<td>Fold sides to centre</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step 06</th>
<th>Step 07</th>
<th>Step 08</th>
<th>Step 09</th>
<th>Step 10</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image6.png" alt="Step 06" /></td>
<td><img src="image7.png" alt="Step 07" /></td>
<td><img src="image8.png" alt="Step 08" /></td>
<td><img src="image9.png" alt="Step 09" /></td>
<td><img src="image10.png" alt="Step 10" /></td>
</tr>
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</thead>
<tbody>
<tr>
<td>Lift both sides and one end of the model so it becomes 3D</td>
<td>Fold flap to centre</td>
<td>Raise end</td>
<td>Fold flap to centre</td>
<td>Complete</td>
</tr>
</tbody>
</table>
The height will be half of the length of the base. To create a box with a different height, modify steps 3 and 5 by making folded sides larger or smaller.

Folding Boxes and Using Data to Create a Model

1. Using your square sheet of paper, fold a box following the instructions given. Note that since our paper is square, the base of the box is also square. (What does this tell you about the length and width of the box?) Measure your box and calculate its volume. What are the units on your answer?

2. Suppose we decrease the height of the box. What do you predict would happen to the length and width of the box? What about the volume of the box?

3. Suppose we increase the height of the box. What do you predict would happen to the length and width of the box? What about the volume of the box?

4. In your group, fold boxes different heights, and calculate the volume of each to fill in the following table. (Two rows are left blank for data for boxes with zero volume.)

<table>
<thead>
<tr>
<th>Height</th>
<th>Length</th>
<th>Width</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.125</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.5</td>
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</tbody>
</table>
5. Enter the data from your table into your calculator to create a scatter plot of the data and sketch it below.

6. What type of model do you choose to approximate your scatter plot? Write the function below and graph your model on the same set of axes with the scatter plot. Does it seem like a reasonable approximation of your data? If not, can you choose a more accurate model?

7. Use your model from question 6 to determine the height should we use to maximize the volume of an origami box. What is the maximum possible volume for your origami box?

Creating a Theoretical Model

8. What is the general formula for the volume of a box with a rectangular base? This formula has too many variables, but already we can eliminate one of them because our box has a square base. What is the modified formula?
9. Unfold the boxes that you made so that we can see the creases. They should look similar to the diagram below. Trace the creases that outline the base of your box and the creases that outline the sides. What is the length of the dashed diagonal lines that have been added?

10. Identify segments of the diagonal lines on the crease diagram that can be used to measure the length and height of the box. Use these segments to find an equation that relates the length of the diagonal to the height, H, and length, L, of the box. (You may want to include the equation \( L + H = \).) How can we use this equation to eliminate another variable from the volume formula?
11. We now have a formula for the volume of an origami box as a function of its height.

(For algebra)

Graph the volume function on your calculator and find the optimal height and maximum volume of an origami box. How does this compare to your model from the data?

(For calculus)

a) Differentiate the volume function and find its critical points.

b) Which critical point is the optimal height of your origami box?

c) What is the maximum volume for your origami box?

d) How do these results compare to your results in question 7?

What if we change the size of the paper that we use?

12. If we change the size of the paper, how would that change the equation that you found in question 10 and the resulting volume function?
13. (For algebra)

Find the optimal height and maximum volume of an origami box folded from a 12” x 12” square sheet of paper.

(For calculus)

Find the optimal height and maximum volume of an origami box folded from a square sheet of paper with sides S inches long. Remember that S is a constant when you are creating and differentiating your volume function, and the input variable is H.

Suppose \( d \) is the length of the diagonal of the square sheet.

What is the length, width, and height of the constructed 3-\( d \) box?
Volume of the box = \( \frac{d^2 d}{16} = \frac{d^3}{128} \)
Let us modify the *masu* box slightly!

\[ a = \frac{d - x}{4} \ldots (1), \quad b = \frac{d - x}{2} \ldots (2), \quad c = \frac{d + 3x}{2} \ldots (3) \]
The volume of the box, $V(x)$ is $abc$.

$$V(x) = \frac{3x^3 - 5dx^2 + d^2 x + d^3}{16}.$$ 

The volume is a function of $x$, note $d$ is a constant.
\[ V(x) = \frac{3x^3 - 5dx^2 + d^2x + d^3}{16} \]

\[ V'(x) = \frac{1}{16} (9x^2 - 10dx + d^2) \]

Note

\[ V'(x) = 0 \]
\[ \Rightarrow 9x^2 - 10dx + d^2 = 0 \]
\[ \Rightarrow x = d, \frac{d}{9} \]

It turns out that the function \( V(x) \) has its maximum at \( x = \frac{d}{9} \), and the maximum value of \( V(x) \) is \( \frac{16d^3}{243} \).
The graph below shows the relationship between the values of $x$ and the volume of the box, $V(x)$, created with a sheet of square paper with a 20-unit long diagonal, that is, $d = 10$.

The $x$ and $y$-coordinates of this point are 1.11, and 65.85, respectively.

The graph of $V(x)$ against the values of $x$.

References